A Hesitant Fuzzy Based Medical Diagnosis Problem

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Abstract. In this paper, we refine the definition of weighted hesitant fuzzy set (WHFS), the concept that allows the membership of a given element is defined in terms of several possible values together with their importance weight, and then introduce some correlation measures for WHFSs. To illustrate the application of proposed correlation measures for WHFSs, we give a practical example in medical diagnosis.

Keywords: Weighted Hesitant Fuzzy Set, Correlation Measure, Medical Diagnosis Problem

1. Introduction

A new generalization of fuzzy set called hesitant fuzzy set (HFS) (Torra (2010)) has received great attention in handling decision making problems where the decision makers have some hesitations among several possible memberships for an element to a set. However, HFS (Torra (2010)) has its inherent drawbacks, because it expresses the membership degrees of an element to a given set only by possible values without emphasizing on the importance of each possible value. In many practical decision making problems, the information provided by decision makers who are familiar with the area might often be described by the same preferences. In such situations, the value repeated several times is more important than that appeared only one time. Thus, the importance of possible membership degrees (i. e., their repetition rate) should be considered in improving the definition of HFS. To consider this fact, Zhang and Wu (2014) introduced the concept of a weighted hesitant fuzzy set, denoted hereafter by (Z-WHFS).

In this contribution, we will show that Zhang and Wu's definition of union, intersection, addition and multiplication operations for Z-WHFS have not been correctly set up. This motivates us to modify and emend a fault of WHFS definition proposed by Zhang and Wu (2014) so as not only the modified definition of WHFS is acceptable in accordance with the well-known axioms for mathematical operations, but also it allows that all information measures are to be defined reasonably as well as those defined for HFSs Farhadinia

(2014a)-Farhadinia (2014e), and Farhadinia and Ban (2013). In this paper, we develop some correlation measures for WHFSs and then, the proposed correlation measures are applied to a medical diagnosis problem.

The present paper is organized as follows: Section 2 introduces some concepts related to WHFSs. Section 3 is presented a number of correlation measures of WHFSs. Section 4 shows the application of correlation measures in medical diagnosis problems. This paper is concluded in Section 5.

2. WHFS Conceptions

Definition 2.1. (Torra (2010)) Let X be a reference set, a HFS A on X is defined in terms of a function $h_A(x)$ when applied to X returns a subset of [0, 1], i. e.

$$A = \{\langle x, h_A(x) \rangle x \in X\},$$

where $h_A(x)$ is a set of some different values in [0, 1], representing the possible membership degrees of the element $x \in X$ to A.

For convenience, we call $h_A(x)$ a hesitant fuzzy element (HFE) (Xia and Xu (2011)) and denoted briefly by h_A.

Assumption 2.1. (See e. g. Farhadinia (2014b) - Xia and Xu (2011)) Notice that the number of values in different HFEs may be different. Suppose that l(h) stands for the number of values in the HFE h. Hereafter, the following assumptions are made: (A1) All the elements in each HFE h are arranged in increasing/decreasing order, and then $h^{\sigma(j)}$ is referred to as the jth largest/smallest value in the HFE h. (A2)

If, for two HFEs $h_1, h_2, L(h_1) \neq l(h_2)$, then $l = \max\{L(h_1), l(h_2)\}$. To have a correct comparison, the two HFEs h_1 and h_2 should have the same length l. If there are fewer elements in h_1 than in h_2, an extension of h_1 should be considered optimistically/pessimistically by repeating its maximum/minimum element until it has the same length with h_2.

Hereafter, we assume that all HFEs have the same length N, and let $h = \cup_{1 \leq j \leq N} \{ h^{\sigma(j)} \}$ throughout the paper.

As can be seen from Definition 2.1, HFS expresses the membership degrees of an element to a given set only by several real numbers between 0 and 1 of equal importance, while in many real-world situations assigning exact values without importance weight to the membership degrees does not describe properly the imprecise or uncertain decision information. Thus, it seems to be difficult for the decision makers to rely on the present form of HFSs for expressing uncertainty of an element. To overcome the difficulty associated with the present form of HFSs, Zhang and Wu (2014) have attempted to introduce the concept of weighted hesitant fuzzy set (Z-WHFS) in which the membership degrees of an element to a given set can be expressed by several possible values together with their importance weight.

Definition 2.2. (Zhang and Wu (2014)) Let X be the universe of discourse. A Zhang and Wu's representation of weighted hesitant fuzzy set (Z-WHFS) on X is defined as

$$w_A = \left\{ \langle x, w_{h_A}(x) \rangle : x \in X \right\} = \left\{ \langle x, \underset{\gamma_A \in w_{h_A}(x)}{\cup} \{ (\gamma_A, w_{x\gamma A}) \} \rangle : x \in X \right\}, \tag{1}$$

where $w_{h_A}(x)$ is a set of some different values in $[0, 1]$, denoting all possible membership degrees of the element $x \in X$ to the set $w_A, w_{x\gamma A} \in [0,1]$ is the weight of γ_A such that $\sum_{\gamma_A \in w_{h_A}(x)} w_{x\gamma A} = 1$ for any $x \in X$.

Zhang and Wu (2014) called $w_{h_A}(x) = \cup_{\gamma_A \in w_{h_A}(x)} \{ (\gamma_A, w_{x\gamma A}) \}$ a weighted hesitant fuzzy element (Z-WHFE). A Z-WHFE is conveniently denoted by $w_{h_A} = \cup_{\gamma_A \in w_{h_A}} \{ (\gamma_A, w_{\gamma_A}) \}$.

Zhang and Wu (2014) defined for three Z-WHFEs $w_h = \cup_{\gamma \in w_h} \{ (\gamma, w_\gamma) \}$, $w_{h_1} = \cup_{\gamma 1 \in w_{h_1}} \{ (\gamma_1, w_{\gamma_1}) \}$ and $w_{h_2} = \cup_{\gamma 2 \in w_{h_2}} \{ (\gamma_2, w_{\gamma_2}) \}$ some operations as follows:

$$w_{h^c} = \underset{\gamma \in w_h}{\cup} \{ (1 - \gamma, w_\gamma) \}; \tag{2}$$

$$w_{h_1} \cup w_{h_2} = \underset{\gamma_1 \in w_{h1}, \gamma_2 \in w_{h2}}{\cup} \{ (\max\{\gamma 1, \gamma 2\}, w_{\gamma_1} \cdot w_{\gamma_2}) \}; \tag{3}$$

$$w_{h_1} \cap w_{h_2} = \underset{\gamma_1 \in w_{h1}, \gamma_2 \in w_{h2}}{\cup} \{ (\min\{\gamma 1, \gamma 2\}, w_{\gamma_1} \cdot w_{\gamma_2}) \}. \tag{4}$$

$$w_{h^\lambda} = \underset{\gamma \in w_h}{\cup} \{ (\gamma^\lambda, w_\gamma) \}; \tag{5}$$

$$\lambda w_h = \underset{\gamma 1 \in w_{h1}, \gamma 2 \in w_{h2}}{\cup} \{ (1 - (1 - \gamma)^\lambda, w_\gamma) \}; \tag{6}$$

$$w_{h1} \oplus w_{h2} = \underset{\gamma_1 \in w_{h1}, \gamma_2 \in w_{h2}}{\cup} \{ (\gamma_1 + \gamma_2 - \gamma_1\gamma_2, w_{\gamma_1} \cdot w_{\gamma_2}) \}; \tag{7}$$

$$w_{h1} \oplus w_{h2} = \underset{\gamma_1 \in w_{h1}, \gamma_2 \in w_{h2}}{\cup} \{ (\gamma_1\gamma_2, w_{\gamma_1} \cdot w_{\gamma_2}) \}. \tag{8}$$

By taking the above mathematical operations into consideration, one can easily find that Zhang and Wu (2014) were careless about their definition of operations because such definitions inherit some fundamental disadvantages (see, Farhadinia (2017)).

Here, consider the Z-WHFE $w_h = \{ \langle 0.7, 0.6 \rangle, \langle 0.8, 0.4 \rangle \}$. Then,

$$w_h \oplus w_h = \{ \langle 0.91, 0.36 \rangle, \langle 0.94, 0.48 \rangle, \langle 0.96, 0.16 \rangle \} \neq 2w_h$$
$$= \{ \langle 0.91, 0.6 \rangle, \langle 0.96, 0.4 \rangle \};$$

$$w_h \oplus w_h = \{ \langle 0.49, 0.36 \rangle, \langle 0.56, 0.48 \rangle, \langle 0.64, 0.16 \rangle \} \neq w_{h^2}$$
$$= \{ \langle 0.49, 0.6 \rangle, \langle 0.64, 0.4 \rangle \}.$$

Here, in order to avoid the disadvantages arising from Zhang and Wu's definition of WHFS and mathematical operations on WHFSs, we redefine a weighted hesitant fuzzy set as follows.

Definition 2.3. Let X be the universe of discourse. A weighted hesitant fuzzy set (WHFS) on X is defined as

$$w_A = \{\langle x, w_{h_A}(x)\rangle : x \in X\} = \left\{\langle x, \underset{1 \le j \le L_x}{\cup}\{\langle h^{\sigma(j)}(x), w_A^{\sigma(j)}(x)\rangle\}\rangle : x \in X\right\}, \qquad (9)$$

Where $w_{h_A}(x)$, referred to as the weighted hesitant fuzzy element (WHFE), is a set of some different values in [0, 1], denoting all possible membership degrees of the element $x \in X$ to the set w_A, $w_A^{\sigma(j)}(x) \in [0,1]$ is the weight of $h_A^{\sigma(j)}(x)$ such that $\sum_{1 \le j \le L_x} w_A^{\sigma(j)}(x) = 1$ for any $x \in X$.

It is interesting to note that if we take $w_A^{\sigma(1)}(x) = \cdots = w_A^{\sigma(L_x)}(x) = \frac{1}{L_x}$ for any $x \in X$, then the WHFS w_A is reduced to a typical HFS.

Hereafter, for the convenience of representation, we denote the WHFE $w_{h_A}(x)$ by $w_{h_A} = \cup_{1 \le j \le L_x}\left\{\langle h_A^{\sigma(j)}, w_A^{\sigma(j)}\rangle\right\}$.

Assumption 2.2. Notice that the number of values in different WHFEs may be different. Suppose that $l(w_{h_1}(x))$ stands for the number of values in $w_{h_1}(x)$. Hereafter, the following assumptions are made: (A1) All the first component of elements in each $w_{h_1}(x)$ are arranged in increasing order, and then $h_1^{\sigma(j)}(x)$ is referred to as the jth largest value in $w_{h_1}(x)$. (A2) If, for some $x \in X$, $l(w_{h_1}(x)) \neq l(w_{h_2}(x))$, then $L_x = \max\left\{l\left(w_{h_1}(x)\right), l(w_{h_2}(x))\right\}$.

To have a correct comparison, the two WHFEs $w_{h_1}(x)$ and $w_{h_2}(x)$ should have the same length L_x. If there are fewer elements in $w_{h_1}(x)$ than in $w_{h_2}(x)$, an extension of $w_{h_1}(x)$ should be considered optimistically by repeating the maximum first component of elements associated with zero weight until it has the same length with $w_{h_2}(x)$ This kind of extension is quite reasonable since the added element with zero weight is meant to be an element that does not really exist.

Throughout this paper, we assume that all WHFEs have the same length N, and let $w_h = \cup_{1 \le j \le N}\left\{\langle h^{\sigma(j)}, w^{\sigma(j)}\rangle\right\}$.

Definition 2.4. Let

$$w_h = \cup_{1 \le j \le N}\left\{\langle h^{\sigma(j)}, w^{\sigma(j)}\rangle\right\}, w_{h_1}$$
$$= \cup_{1 \le j \le N}\left\{\langle h_1^{\sigma(j)}, w_1^{\sigma(j)}\rangle\right\} \text{and } w_{h_2}$$
$$= \cup_{1 \le j \le N}\left\{\langle h_2^{\sigma(j)}, w_2^{\sigma(j)}\rangle\right\}$$

be three WHFEs. Then, some operations on the WHFEs w_h, w_{h_1} and w_{h_2} are defined as the following:

$$w_{h^c} = \underset{1 \le j \le N}{\cup}\{\langle 1 - h^{\sigma(j)}, w^{\sigma(j)}\rangle\}; \qquad (10)$$

$$w_{h_1} \cup w_{h_2} = \underset{1 \le j \le N}{\cup}\left\{\langle \max\{h_1^{\sigma(j)}, h_2^{\sigma(j)}\}, \overline{(w_1^{\sigma(j)} + w_2^{\sigma(j)})}\rangle\right\}; \qquad (11)$$

$$w_{h_1} \cap w_{h_2} = \underset{1 \le j \le N}{\cup}\left\{\langle \min\{h_1^{\sigma(j)}, h_2^{\sigma(j)}\}, \overline{(w_1^{\sigma(j)} + w_2^{\sigma(j)})}\rangle\right\}; \qquad (12)$$

$$w_{h^\lambda} = \underset{1 \le j \le N}{\cup}\left\{\langle h^{\sigma(j)\lambda}, w^{\sigma(j)}\rangle\right\}; \qquad (13)$$

$$\lambda w_h = \underset{1 \le j \le N}{\cup}\{\langle 1 - (1 - h^{\sigma(j)})^\lambda, w^{\sigma(j)}\rangle\}; \qquad (14)$$

$$w_{h1} \oplus w_{h2} = \underset{1 \le j \le N}{\cup}\left\{\langle h_1^{\sigma(j)} + h_2^{\sigma(j)} - h_1^{\sigma(j)}h_2^{\sigma(j)}, \overline{(w_1^{\sigma(j)} + w_2^{\sigma(j)})}\rangle\right\}; \qquad (15)$$

$$w_{h1} \oplus w_{h2} = \underset{1 \le j \le N}{\cup}\left\{\langle h_1^{\sigma(j)} h_2^{\sigma(j)}, \overline{(w_1^{\sigma(j)} + w_2^{\sigma(j)})}\rangle\right\},$$

In the above formulas, $\overline{(w_1^{\sigma(j)} + w_2^{\sigma(j)})}$ for $1 \le j \le N$, referred to as the normalized weights, are determined in two steps: (i) We first calculate the weight of jth component of the binary operation $w_{h1} \odot w_{h2}$ by simply adding the weights $w_1^{\sigma(j)}$ and $w_2^{\sigma(j)}$ for $1 \le j \le N$; (ii) After the whole components of $w_{h1} \odot w_{h2}$ are to be obtained, their weights are considered again and then normalized.

Example 2.1. Suppose that
$w_{h1} = \{\langle 0.2, 0.1\rangle, \langle 0.4, 0.3\rangle, \langle 0.5, 0.6\rangle\}$ and $w_{h1} = \{\langle 0.3, 0.5\rangle, \langle 0.7, 0.5\rangle\}$ are two given WHFEs. Bearing Assumption 2.1 in mind, w_{h1} should be first extended as $w_{h2} = \{\langle 0.3, 0.5\rangle, \langle 0.7, 0.5\rangle, \langle 0.7, 0.0\rangle\}$. Then, one gets

$$w_{h_1^c} = \{\langle 0.5, 0.6\rangle, \langle 0.6, 0.3\rangle, \langle 0.8, 0.1\rangle\};$$

$$w_{h1} \cup w_{h2} = \left\{ \langle \max\{0.2, 0.3\}, \frac{(0.1 + 0.5)}{2} \rangle, \langle 0.7, 0.4 \rangle, \langle 0.7, 0.3 \rangle \right\};$$

$$w_{h1} \cap w_{h2} = \left\{ \langle \min\{0.2, 0.3\}, \frac{(0.1 + 0.5)}{2} \rangle, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.3 \rangle \right\};$$

$$(w_{h1} \cup w_{h2}) \cup w_{h1} = \left\{ \langle \max\{\max\{0.2, 0.3\}, 0.2\}, \frac{((0.1 + 0.5) + 0.1)}{3} \rangle, \langle 0.7, \frac{1.1}{3} \rangle, \langle 0.7, \frac{1.2}{3} \rangle \right\};$$

$$(w_{h1} \cap w_{h2}) \cap w_{h1} = \left\{ \langle \min\{\min\{0.2, 0.3\}, 0.2\}, \frac{((0.1 + 0.5) + 0.1)}{3} \rangle, \langle 0.4, \frac{1.1}{3} \rangle, \langle 0.5, \frac{1.2}{3} \rangle \right\};$$

$$w_{h_1^\lambda} = \left\{ \langle 0.2^\lambda, 0.1 \rangle, \langle 0.4^\lambda, 0.3 \rangle, \langle 0.5^\lambda, 0.6 \rangle \right\};$$

$$\lambda w_{h1} = \left\{ \langle 1 - 0.8^\lambda, 0.1 \rangle, \langle 1 - 0.6^\lambda, 0.3 \rangle, \langle 1 - 0.5^\lambda, 0.6 \rangle \right\};$$

$$w_{h1} \oplus w_{h2} = \left\{ \langle 0.44, 0.3 \rangle, \langle 0.82, 0.4 \rangle, \langle 0.85, 0.3 \rangle \right\};$$

$$w_{h1} \otimes w_{h2} = \left\{ \langle 0.06, 0.3 \rangle, \langle 0.28, 0.4 \rangle, \langle 0.35, 0.3 \rangle \right\}.$$

Theorem 2.1. Farhadinia (2017) Let
$w_h = \cup_{1 \le j \le N} \left\{ \langle h^{\sigma(j)}, w^{\sigma(j)} \rangle \right\}, w_{h1} = \cup_{1 \le j \le N} \left\{ \langle h_1^{\sigma(j)}, w_1^{\sigma(j)} \rangle \right\}$ and $w_{h2} = \cup_{1 \le j \le N} \left\{ \langle h_2^{\sigma(j)}, w_2^{\sigma(j)} \rangle \right\}$ be three WHFEs. Then, all operations $w_{h_1^c}, w_{h1} \cup w_{h2}, w_{h1} \cap w_{h2}, w_{h_1^\lambda}, \lambda w_{h1}, w_{h1} \oplus w_{h2}, w_{h1} \otimes w_{h2}$ given in Definition 2.4 are also WHFEs.

Theorem 2.2. Farhadinia (2017) Let

$$w_h = \cup_{1 \le j \le N} \left\{ \langle h^{\sigma(j)}, w^{\sigma(j)} \rangle \right\}, w_{h1} = \cup_{1 \le j \le N} \left\{ \langle h_1^{\sigma(j)}, w_1^{\sigma(j)} \rangle \right\}$$

and $w_{h2} = \cup_{1 \le j \le N} \left\{ \langle h_2^{\sigma(j)}, w_2^{\sigma(j)} \rangle \right\}$ be three WHFEs. Then,

$$\left(w_{h^c} \right)^\lambda = (\lambda w_h)^c \; ; \quad \left(w_{h^\lambda} \right)^c = \lambda(w_{h^c}) \; ; \tag{16}$$

$$(w_{h1} \cup w_{h2})^c = w_{h_1^c} \cap w_{h_2^c} \; ; \quad (w_{h1} \cap w_{h2})^c = w_{h_1^c} \cup w_{h_2^c} \; ; \tag{17}$$

$$(w_{h1} \otimes w_{h2})^c = w_{h_1^\lambda} \otimes w_{h_2^\lambda} \; ; \quad \lambda(w_{h1} \oplus w_{h2}) = \lambda w_{h_1} \oplus \lambda w_{h_2} \; ; \tag{18}$$

$$(w_{h1} \oplus w_{h2})^c = w_{h_1^c} \otimes w_{h_2^c} \; ; \quad (w_{h1} \otimes w_{h2})^c = w_{h_1^c} \oplus w_{h_2^c} \; ; \tag{19}$$

$$w_{h1} \oplus w_{h2} = w_{h_2} \oplus w_{h_1} \; ; \quad w_{h1} \otimes w_{h2} = w_{h2} \otimes w_{h1} \; ; \tag{20}$$

$$w_{h1} \cup w_{h2} = w_{h2} \cup w_{h1} \; ; \quad w_{h1} \cap w_{h2} = w_{h2} \cap w_{h1} \; ; \tag{21}$$

$$w_h \cup (w_{h1} \cup w_{h2}) = (w_h \cup w_{h1}) \cup w_{h2} \; ; \quad w_h \cap (w_{h1} \cap w_{h2}) = (w_h \cap w_{h1}) \cap w_{h2} \tag{22}$$

$$w_h \cup w_h = w_h \; ; \quad w_h \cap w_h = w_h \; ; \tag{23}$$

$$w_h \oplus w_h = 2w_h \; ; \quad w_h \otimes w_h = w_{h^2} \tag{24}$$

3. Correlation Measures for WHFSs

Definition 3.1. A real-valued function ρ is called a correlation measure for WHFSs, if for WHFSs w_A, w_B on X, ρ satisfies the following properties:

$$(\rho 1) \quad 0 \le \rho(w_A, w_B) \le 1 \; ;$$

$$(\rho 2) \quad \rho(w_A, w_B) = \rho(w_B, w_A);$$

$$(\rho 3) \quad \rho(w_A, w_B) = 1 \; if \; w_A = w_B.$$

By assuming that

$$w_A = \{\langle x, w_{h_A}(x)\rangle : x \in X\} = \left\{\langle x, \underset{1 \le j \le N}{\cup} \left\{\langle h_A^{\sigma(j)}(x), w_A^{\sigma(j)}(x)\rangle\right\}\rangle : x \in X\right\}, \tag{25}$$

$$w_B = \{\langle x, w_{h_B}(x)\rangle : x \in X\} = \left\{\langle x, \underset{1 \le j \le N}{\cup} \left\{\langle h_B^{\sigma(j)}(x), w_B^{\sigma(j)}(x)\rangle\right\}\rangle : x \in X\right\}, \tag{26}$$

we define the following correlation measure formulas for any two WHFSs w_A and w_B as

$$\rho w\,H\,F\,S1(w_A, w_B) = \frac{\sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N} w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i)\, w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N}(w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i))^2\right)}\sqrt{\sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N}(w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i))^2\right)}}, \tag{27}$$

In order to equip the WHFS theory with further correlation measures, we presented two other correlation measures for WHFSs by extending Jaccard (1901)'s and Dice (1945)'s correlation measures defined on the vector space as follows:

$$\rho w\,H\,F\,S2(w_A, w_B) = \left\{\sum_{i=1}^{n}(\frac{1}{N}\sum_{i=1}^{n} w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i)\, w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i))\right\}/$$

$$\sum_{i=1}^{n}(\frac{1}{N}\sum_{j=1}^{N}\left(w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i)\right)^2 - \sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N} w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i)\, w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i)\right)\Bigg\}, \tag{28}$$

$$\left\{\sum_{i=1}^{n}(\frac{1}{N}\sum_{j=1}^{N}\left(w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i)\right)^2 + \right.$$

$$\rho w\,H\,F\,S3(w_A, w_B) = \frac{2\sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N} w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i)\, w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i)\right)}{\sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N}(w_A^{\sigma(j)}(x_i)\, h_A^{\sigma(j)}(x_i))^2\right) + \sum_{i=1}^{n}\left(\frac{1}{N}\sum_{j=1}^{N}(w_B^{\sigma(j)}(x_i)\, h_B^{\sigma(j)}(x_i))^2\right)}. \tag{29}$$

Theorem 3.1. Farhadinia (2017) The measure functions $\rho WHFSi((w_A, w_B)$ $(i = 1,2,3)$ given respectively by (27)-(29) are correlation measures for WHFSs w_A and w_B.

4. WHF Information Used in Medical Diagnoses

In this portion, we implement the following medical diagnosis problem to illustrate the efficiency of the correlation measures for WHFSs.

Example 4.1. Consider the set of diagnoses D = {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}. The aim here is to assign a patient with the given values of the symptoms, S = {Temperature, Headache, Cough, Stomach pain, Chest pain} to one of the aforementioned diagnoses. Three medical experts El, (l = 1, 2, 3) are invited to provide their possible assessment of diagnoses with respect to symptoms. For each diagnosis with respect to each symptom, all of the medical experts provide anonymously their evaluated values. As an example, for the diagnosis "Viral fever" with respect to the symptom "Temperature", the evaluation value provided by

medical experts E1 and E3 is 0.5; and E2's evaluation value is 0.7. In this regard, and noting that the weights of three medical experts are unknown, the evaluation of "Viral fever" with respect to "Temperature" can be represented by a WHFE as

$$w_h\,(V\,iralfever, Temperature) = w_{h11} = \left\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\right\}.$$

Note that the characteristics of the diagnosis "Viral fever" with respect to the symptoms "Headache", "Cough", "Stomach pain", "Chest pain", denoted respectively by WHFEs w_{h1j}, (j = 2, 3, 4, 5), form the WHFS w_{h1} which is indicated in the first row of Table 1. The results evaluated for other diagnoses with respect to symptoms arecontained in a weighted hesitant fuzzy decision matrix, shown in Table 1.

Furthermore, suppose that the set of patients is P = {Al, Bob, Joe, Ted}, and the symptoms characteristic for the considered patients are evaluated and given by the three medical experts in the form of a weighted hesitant fuzzy matrix demonstrated in Table 2. Here, the main task is to seek a diagnosis for each patient.

Table 1. Symptoms characteristic for the considered diagnoses.

	Temperature	Headache	Cough	Stomach pain	Chest pain
Viral fever	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.3, \frac{3}{3}\rangle\}$	$\{\langle 0.4, \frac{1}{3}\rangle, \langle 0.6, \frac{2}{3}\rangle\}$	$\{\langle 0.3, \frac{2}{3}\rangle, \langle 0.4, \frac{1}{3}\rangle\}$	$\{\langle 0.6, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$
Malaria	$\{\langle 0.4, \frac{1}{3}\rangle, \langle 0.7, \frac{2}{3}\rangle\}$	$\{\langle 0.5, \frac{1}{3}\rangle, \langle 0.8, \frac{2}{3}\rangle\}$	$\{\langle 0.5, \frac{3}{3}\rangle\}$	$\{\langle 0.2, \frac{2}{3}\rangle, \langle 0.3, \frac{1}{3}\rangle\}$	$\{\langle 0.7, \frac{1}{3}\rangle, \langle 0.8, \frac{2}{3}\rangle\}$
Typhoid	$\{\langle 0.3, \frac{2}{3}\rangle, \langle 0.4, \frac{1}{3}\rangle\}$	$\{\langle 0.5, \frac{1}{3}\rangle, \langle 0.6, \frac{2}{3}\rangle\}$	$\{\langle 0.1, \frac{2}{3}\rangle, \langle 0.4, \frac{1}{3}\rangle\}$	$\{\langle 0.6, \frac{3}{3}\rangle\}$	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$
Stomach problem	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.7, \frac{3}{3}\rangle\}$	$\{\langle 0.4, \frac{2}{3}\rangle, \langle 0.5, \frac{1}{3}\rangle\}$	$\{\langle 0.5, \frac{1}{3}\rangle, \langle 0.8, \frac{2}{3}\rangle\}$
Chest problem	$\{\langle 0.2, \frac{2}{3}\rangle, \langle 0.3, \frac{1}{3}\rangle\}$	$\{\langle 0.4, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.4, \frac{1}{3}\rangle, \langle 0.6, \frac{2}{3}\rangle\}$	$\{\langle 0.3, \frac{2}{3}\rangle, \langle 0.4, \frac{1}{3}\rangle\}$	$\{\langle 0.4, \frac{3}{3}\rangle\}$

Table 2. Symptoms characteristic for the considered patients.

	Temperature	Headache	Cough	Stomach pain	Chest pain
Al	$\{\langle 0.4, \frac{3}{3}\rangle\}$	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.6, \frac{1}{3}\rangle, \langle 0.7, \frac{2}{3}\rangle\}$	$\{\langle 0.2, \frac{2}{3}\rangle, \langle 0.4, \frac{1}{3}\rangle\}$	$\{\langle 0.1, \frac{1}{3}\rangle, \langle 0.2, \frac{2}{3}\rangle\}$
Bob	$\{\langle 0.6, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.5, \frac{1}{3}\rangle, \langle 0.8, \frac{2}{3}\rangle\}$	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.6, \frac{1}{3}\rangle\}$	$\{\langle 0.3, \frac{3}{3}\rangle\}$	$\{\langle 0.4, \frac{2}{3}\rangle, \langle 0.5, \frac{1}{3}\rangle\}$
Joe	$\{\langle 0.2, \frac{2}{3}\rangle, \langle 0.3, \frac{1}{3}\rangle\}$	$\{\langle 0.5, \frac{3}{3}\rangle\}$	$\{\langle 0.2, \frac{2}{3}\rangle, \langle 0.4, \frac{1}{3}\rangle\}$	$\{\langle 0.6, \frac{1}{3}\rangle, \langle 0.7, \frac{2}{3}\rangle\}$	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$
Ted	$\{\langle 0.4, \frac{3}{3}\rangle\}$	$\{\langle 0.4, \frac{2}{3}\rangle, \langle 0.7, \frac{1}{3}\rangle\}$	$\{\langle 0.3, \frac{1}{3}\rangle, \langle 0.4, \frac{2}{3}\rangle\}$	$\{\langle 0.7, \frac{2}{3}\rangle, \langle 0.8, \frac{1}{3}\rangle\}$	$\{\langle 0.5, \frac{2}{3}\rangle, \langle 0.6, \frac{1}{3}\rangle\}$

As can be seen from Tables 1 and 2, all WHFEs are not in the same size. To circumvent this issue, we implement Assumption 2.6. In this regard, The WHFEs with fewer elements are extended optimistically by repeating the maximum first component of elements associated with zero weight until it has the same length with others. For example, the WHFE w_h (Viralfever, Temperature) $:= w_{h12} = \{\langle 0.3, \frac{3}{3}\rangle\}$ is extended to $\{\langle 0.3, \frac{3}{3}\rangle, \langle 0.3, 0\rangle\}$.

During the process of deriving a diagnosis for each patient, the degree of dependence between the rows of Tables 1 and 2 should be analyzed. For instance, the first rows of Tables 1 and 2 which are regarded as the following two WHFSs

$$Al = \{hTemperature, \{\langle 0.4, \tfrac{3}{3}\rangle, \langle 0.4, 0\rangle\}, \langle Headache, \{\langle 0.5, \tfrac{2}{3}\rangle, \langle 0.7, \tfrac{1}{3}\rangle\}\rangle, \langle Cough, \{\langle 0.6, \tfrac{1}{3}\rangle, \langle 0.7, \tfrac{1}{3}\rangle\}\rangle,$$

$$\langle Stomachpain, \{\langle 0.2, \tfrac{2}{3}\rangle, \langle 0.4, \tfrac{1}{3}\rangle\}\rangle, \langle Chestpain, \{\langle 0.1, \tfrac{1}{3}\rangle, \langle 0.2, \tfrac{2}{3}\rangle\}\rangle;$$

$$Viralfever = \left\{\begin{array}{l} \langle Temperature, , \{\langle 0.5, \tfrac{2}{3}\rangle, \langle 0.7, \tfrac{1}{3}\rangle\}\rangle, \langle Headache, \{\langle 0.3, \tfrac{3}{3}\rangle, \langle 0.3, 0\rangle\}\rangle, \langle Cough, \{\langle 0.3, \tfrac{1}{3}\rangle, \langle 0.6, \tfrac{2}{3}\rangle\}\rangle, \\ \langle Stomachpain, \{\langle 0.3, \tfrac{2}{3}\rangle, \langle 0.4, \tfrac{1}{3}\rangle\}\rangle, \langle Chestpain, \{\langle 0.6, \tfrac{2}{3}\rangle, \langle 0.7, \tfrac{1}{3}\rangle\}\rangle \end{array}\right\},$$

are taken into account to determine the correlated degree of Al and viral fever.

In order to proceed, we apply the correlation measure $\rho WHFS1$ to determine the degree of dependence between diagnoses and patients. The results obtained by the use of these correlation measures are shown in Table 3.

Table 3. Values of $\rho WHFS1$ for each patient to the considered set of possible diagnoses.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest Problem
Al	0.7984	0.5376	0.5998	0.6467	0.7906
Bob	0.7261	0.7581	0.8549	0.7881	0.7781
Joe	0.8030	0.6568	0.5697	0.6706	0.7356
Ted	0.8513	0.5778	0.8723	0.6798	0.8433

By comparing the results listed in Table 3, we observe that AL and Joe suffer from "Viral fever", Bob and Ted from "Typhoid".

Table 3 present the same result for all correlation measures.

5. Conclusion

In this contribution, we modified and emended a fault of WHFS definition so as it is acceptable in accordance with the well-known axioms for mathematical operations. We believe many future works can be developed by the use of the findings of this contribution which support the decision makers in making decisions effectively in WHFS-structured MAGDM problems.

Acknowledgements

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